OBERSEMINAR ARAKELOVTHEORIE: EQUIDISTRIBUTION OF WEIERSTRASS POINTS

Organizers: Philipp Jell and Florent Martin
Time: Tuesday 14–16 h
SS 2016/17 Room M102

Contents

Given a smooth projective curve X over a field K, the classical *Weierstrass points* are points for which the set of global sections of $\mathcal{O}(nP)$ shows a very special behaviour.

More generally, one can associate to a line bundle $\mathcal{L}$ on X a notion of Weiterstrass points. The classical Weiterstrass points are then the ones where $\mathcal{L} = \Omega^1$.

In the seminar, we study the distribution of the Weiterstrass points W_m of $\mathcal{L}^m$ as $m \to \infty$. If the field K is the field of complex numbers $\mathbb{C}$, there is a convergence result;

$$\lim_{m\to\infty} \frac{1}{C_m} \sum_{w\in W_m} \delta_w$$

to the so called Arakelov Bergman measure 1 (where C_m is some normalization factor).

We want to study an analogue of this result in non-archimedean geometry, proved by Amini [Ami14].

We will cover the use of classical result (2) and the classical results by Neeman and Mumford (3). To cover the analogue over non-archimedean fields, we will give an introduction to Zhang's measure (4) and limit linear series (5).

Beware that [1] is not online at the moment.

Date: April 24, 2017.

1. Introduction (25.04., Philipp Jell)

Present an introduction to the topic of Weierstrass points. Give the defintion both of classical Weierstrass points as well the Weiterstrass points of a line bundle. State the main result of [Ami14] as well as its classical analogue.

2. Classical Weierstrass points (02.05, Bastian Altmann)

The goal of this talk is to define classical Weiterstrass points for curves over $\mathbb{C}$ as well as the notion of Weiterstrass points of a line bundle.

Define Weierstrass points as in [FK, III.5] and discuss their relation the the Riemann-Roch and the Weierstrass-gap-theorem. Show that the definition there agrees with the one in [Ami14, 1.1] (for $L = \Omega^1$).

To illustrate the use of Weierstrass points, show how they are used to proof the finiteness of the automorphism group of a curve of genus $g > 1$ [FK, V.I.I & V.I.II].

You do not need the case of positive characteristic from [Ami14, 1.1].

3. Results by Neeman and Mumford over $\mathbb{C}$ (09.05, Jascha Smacka)

Explain Mumford's result (the one mentioned in the first paragraph of [Ami14]). One should first give a precise definition of what is called the Arakelov-Bergmann measure in [Ami14] (what Neeman calls the Bergmann measure in [Nee84, Section 3]), and then state Mumford-Neeman equidistribution result and expose the ideas of the proof in [Nee84].

4. Zhang's measure (16.05)

Recall (very) briefly the material in [Ami14, Section 1.2] on non-archimedean curves. Also define metrized complex of κ-curve and explain why the skeleton Γ is a metrized complex of κ-curves [AB, section 1.2]. Then explain carefully [Ami14, Section 1.3] (looking at [33] and give ideas of the proofs would be worthwhile).

5. Background on limit linear series (23.05)

Define the notion of a divisor on a metrized complex of curves [AB, Section 2.1]. Give the proof of [AB, Lemma 4.3] (which is [Ami14, Lemma 2.2]) and explain [AB, Theorem 4.5 & Theorem 4.6].

6. Slope structures (30.05)

Give the definitions from [Ami14, Section 2.1]. It will be important to illustrate these definitions with easy examples, to give the audience some kind of intuition of what is going on.

7. Limit linear series (13.06)

The goal of this talk is to explain [Ami14, Section 2.2]. Since the main source is not available (yet), explain carefully the definitions made there as well as [AB, Theorem 5.9] and its proof.

8. Reduction of Weierstrass point in characteristic 0 (20.06)

Following [Ami14, Section 3.1], explain the proof of [Ami14, Theorem 1.5]. We don't need to cover [Ami14, Section 3.2].

9. An introduction to Okounkov bodies (27.06)

Given a line bundle L on a projective variety X, one can associate its *volume* which measures the asymptotic growth of $h^0(X, L^{\otimes m})$. This volume is related to a certain convex polytope: a so-called Okounkov body. The goal of this talk is to explain and prove the fundamental result in this theory of Okounkov bodies, namely [LM09, Theorem A] (the proof is given in [LM09, Theorem 2.3]). In order to do so, one has to cover section 1, subsection 2.1 and subsection 2.2 of [LM09].

10. Local equidistribution (04.07)

Explain the results of [Ami14, Section 4.1]. One should give a detailed proof of [Ami14, Theorem 4.1] following the presentation of [Bou14]. Namely, one should present [Bou14, Theorem 0.2] and its proof, and for that, one should cover [Bou14, Section 1] stopping after [Bou14, Corollary 1.14]. Beware that [Bou14] is written in French.

11. Proof of the main theorem (11.7)

The goal of this talk is to cover [Ami14, Section 5] and hence to prove Theorem 1 and Theorem 2 of [Ami14].

References

[AB] O. Amini and Matt Baker. Linear series on metrized complexes of algebraic curves. `https://arxiv.org/abs/1204.3508`.

[Ami14] O. Amini. Equidistribution of Weierstrass points on curves over non-Archimedean fields. *ArXiv e-prints*, December 2014.

[Bou14] Sébastien Boucksom. Corps d'Okounkov (d'après Okounkov, Lazarsfeld-Mustaţă et Kaveh-Khovanskii). *Astérisque*, (361):Exp. No. 1059, vii, 1–41, 2014.

[FK] Hershel M. Fraskas and Irwin Kra. *Riemann Surfaces.*

[LM09] Robert Lazarsfeld and Mircea Mustaţă. Convex bodies associated to linear series. *Ann. Sci. Éc. Norm. Supér. (4)*, 42(5):783–835, 2009.

[Nee84] Amnon Neeman. The distribution of Weierstrass points on a compact Riemann surface. *Ann. of Math. (2)*, 120(2):317–328, 1984.