OBERSEMINAR ARAKELOVTHEORIE: NORMS AND METRICS IN NON-ARCHIMEDEAN GEOMETRY

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Contents

Let K be a non-Archimedean field, X a smooth projective variety over K and L an ample line bundle on X. The goal of this seminar is to study some notions of positive metrics on L, Monge-Ampère measures and Monge-Ampère energy associated to metrics on L. These objects are related to non-Archimedean pluripotential theory. Our goal is to understand some connections between the above quantities and questions related to small sections of L with respect to metrics on L, and equidistribution of Fekete points.

We will follow for that the preprint [BE18] *Norms, determinant of cohomology and Fekete points in non-Archimedean geometry* from Sébastien Boucksom and Denis Eriksson.

The paper [BE18] gives results in a uniform way for non-Archimedean, Archimedean and trivially valued fields. In this seminar we'll focus only on the non-Archimedean case. Hence in the talks, statement, proofs, examples and so on should focus on the non-Archimedean case. Moreover if proofs or construction can be simplified by assuming that K° is Noetherian, then this assumption should be made.

1. Norms on non-Archimedean vector spaces (17.04.18, Stefan Stadlöder)

The goal of this talk is to cover the section 1 *Space of norms* of [BE18] taking care only of the non-Archimedean case (and one should ignore the trivially valued case). The main point is to study ultrametric norms on finite dimensional K-vector spaces V, equip them with norms (subsection 1.2), and study the class of diagonalizable norms (subsection 1.3), and show some density result for those norms (subsection 1.4). One other important point will be to explain the construction of the norm $\|\cdot\|_{\mathcal{V}}$ induced by a lattice $\mathcal{V} \subset V$ (subsection 1.7).

2. Determinants and successive minima (24.04.18)

Cover the section 2 *Determinants and successive minima* of [BE18]. One should only cover the non-Archimedean case.

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3. Analytifications, models and the reduced fiber theorem (08.05.18)

Cover the section 4 *Analytification and models* of [BE18]: after introducing Berkovich analytifications, one should introduce models of K-varieties, their reductions, and explain the special case of the reduced fiber Theorem (Theorem 4.20 of [BE18]).

4. Metrics on non-Archimedean line bundles, and Fubini-Study metrics (15.05.18)

Cover the section 5 *Metrics* of [BE18] which introduces many notions around metrics on line bundles over non-Archimedean varieties, in particular Fubini-Study metrics.

5. Limits of Fubini-Study metrics and semipositive envelopes (29.05.18)

Cover section 6 *Limits of Fubini-Study metrics* of [BE18]. The goal is to introduce and study many classes of metrics on line bundles in the non-Archimedean case: asymptotically Fubini-Study, semipositive, smooth psh, psh-regularizable, globally psh-approachable metrics (subsection 6.1 of [BE18]). Then one should introduce and study the operators FS_m and P_m which map norms on the space of global sections to certain metrics (subsection 6.2). Finally one should use the above results to construct the semipositive envelope $\mathrm{P}(\phi)$ of a bounded metric (one could decide to skip conjectures 6.20, 6.21 and Lemma 6.22). One should only focus on the non-Archimedean case.

6. Polynomial maps, determinants, functors (05.06.18, Thomas Fenzl)

Cover the first part of the appendix (A.1, A.2, A.3, A.4, A.5) of [BE18] which will be necessary for the constructions of the next talk.

7. Determinant of cohomology and Deligne pairings (12.06.18)

The goal of this talk is to introduce the determinant of cohomology associated to a proper flat morphism, then the Deligne pairings $\langle L_0, \dots, L_n \rangle$ and the Knudsen-Mumford expansion. For this talk, one should always make the simplifying assumptions that schemes are Noetherian, and moreover, the assumption "X admissible" should be replaced by the simplifying assumption "K is a discretely valued non-Archimedean field, and X is a scheme of finite type over K°". Cover subsections A.6, A.7 and A.8 from the appendix of [BE18].

8. Monge-Ampère measures and Deligne pairings (19.06.18)

The goal of this talk is twofold.

First one should explain the construction of mixed Monge-Ampère measures following subsection 7.1 of [BE18] (there you can ignore the Bedford-Taylor theory in the complex case) and give the properties given in subsection 7.1 of [BE18].

Then, one should explain the construction of the metric on the Deligne pairing $\langle L_0, \dots, L_n \rangle$ associated with some models $\mathcal{L}_0 \dots \mathcal{L}_n$. For this, cover 7.2 of [BE18]. One does not necessarily needs to state Theorem 7.9.

9. Relation between the Deligne pairings and mixed Monge-Ampère measures (26.06.18)

The goal of this talk is to prove the Theorem 7.9 of [BE18] which gives a precise relation between mixed Monge-Ampère measures and associated metrics on the Deligne pairings. For this, cover the subsection 7.3 of [BE18].

10. Asymptotics of relative volumes (03.07.18)

The goal of this talk is explain the relation between on the one hand non-Archimedean volumes associated to metrics ϕ and ψ and on the other hand the relative Monge-Ampère energy $E(P(\phi), P(\psi))$ defined in terms of mixed Monge-Ampère measures. Cover the Section 8 *Asymptotics of relative volumes* of [BE18] focusing on the non-Archimedean case. This means covering the subsections 8.1, 8.2, 8.4, 8.5.

11. Transfinite diameters (?)

The goal of this talk is to explain the notion of transfinite diameter $\delta_{\infty,\psi}(\phi)$ associated to continuous metrics ψ, ϕ on an ample line bundle L on X, and the associated notion of Fekete configuration. The main result is then an equidistribution result (Theorem 9.11) of these Fekete configurations relating asymptotically the Fekete configurations, Monge-Ampère measure and semipositive envelopes. Cover subsections 9.1 and 9.2 of [BE18].

References

[BE18] Sébastien Boucksom and Denis Eriksson. Norms, determinant of cohomology and Fekete points in non-Archimedean geometry. *preprint*, 2018.